It suffices to show that, for any $a_i \in I$, $1 + a_1t + \cdots + a_nt^n$ is invertible in $R[t]$. Let $\sum_i b_it^i$ be the "formal" inverse of $1 + a_1t + \cdots + a_nt^n$ in the power series ring $R[[t]]$. Then we have: $b_0 = 1$, $b_1 = -a_1$, $b_2 = -(b_1a_1 + a_2)$, $b_3 = -(b_2a_1 + b_1a_2 + a_3)$, etc. Let A be the companion matrix.
By direct matrix multiplication, we see that, for $e = (0, \ldots, 0, 1)$:
$eA = (0, \ldots, 1, -a_1) = (0, \ldots, 0, b_0, b_1)$
$eA^2 = (0, \ldots, 0, b_0, b_1, -b_0a_2 - b_1a_1) = (0, \ldots, 0, b_0, b_1, b_2)$,
$\cdots\cdots$
$eA^n = (b_1, \ldots, b_{n-1}, b_n)$
$eA^{n+1} = (b_2, \ldots, b_n, b_{n+1})$, etc.
Also, we can check that the entries of A^n "involve" only the a_i's and not the element 1, so $A^n \in M_n(I)$. By the hypothesis, A^n is nilpotent, so the equations for eA^k above show that $b_k = 0$ for sufficiently large k. Therefore, the formal inverse for $1 + \sum_i a_it^i$ lies in $R[t]$, as desired.