$$tanA + \tan\left(A + \frac{\pi}{3}\right) + \tan\left(A + \frac{2\pi}{3}\right) = \tan A + \tan\left(A + \frac{\pi}{3}\right) + \tan\left(\pi - \left(\frac{\pi}{3} - A\right)\right) =$$

$$= \tan A + \tan\left(A + \frac{\pi}{3}\right) - \tan\left(\frac{\pi}{3} - A\right) = \tan A + \tan\left(A + \frac{\pi}{3}\right) + \tan\left(A - \frac{\pi}{3}\right) =$$

$$= \tan A + \frac{\tan A + \tan\frac{\pi}{3}}{1 - \tan A \tan\frac{\pi}{3}} + \frac{\tan A - \tan\frac{\pi}{3}}{1 + \tan A \tan\frac{\pi}{3}} =$$

$$= \tan A + \frac{\tan A + \sqrt{3}}{1 - \sqrt{3}\tan A} + \frac{\tan A - \sqrt{3}}{1 + \sqrt{3}\tan A}$$

$$= \tan A + \frac{\tan A + \sqrt{3} + \sqrt{3}\tan^2 A + 3\tan A + \tan A - \sqrt{3} - \sqrt{3}\tan^2 A + 3\tan A}{1 - 3\tan^2 A} =$$

$$= \tan A + \frac{\tan A \quad +3\tan A + \tan A + 3\tan A}{1 - 3\tan^2 A} = \tan A + \frac{8\tan A}{1 - 3\tan^2 A} = \tan A\left(1 + \frac{8}{1 - 3\tan^2 A}\right) =$$

$$= \tan A\left(\frac{1 - 3\tan^2 A + 8}{1 - 3\tan^2 A}\right) = 3\left(\frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}\right) = 3tg(3A) \neq 3$$

I've verify this result in Mathcad:

$$\tan(x) + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right)$$

Simplify :=

$$3 \cdot \tan(3 \cdot x)$$

It's true!

So, $\tan A + \tan(A + \frac{\pi}{3}) + \tan(A + \frac{2\pi}{3}) \neq 3$

$$\tan A + \frac{\tan A + \tan\frac{\pi}{3}}{1 - \tan A \tan\frac{\pi}{3}} + \frac{\tan A + \tan\frac{2\pi}{3}}{1 - \tan A \tan\frac{2\pi}{3}} == \tan A + \frac{\tan A + \sqrt{3}}{1 - \sqrt{3}\tan A} + \frac{\tan A + \tan\frac{2\pi}{3}}{1 - \tan A \tan\frac{2\pi}{3}}$$