Task:
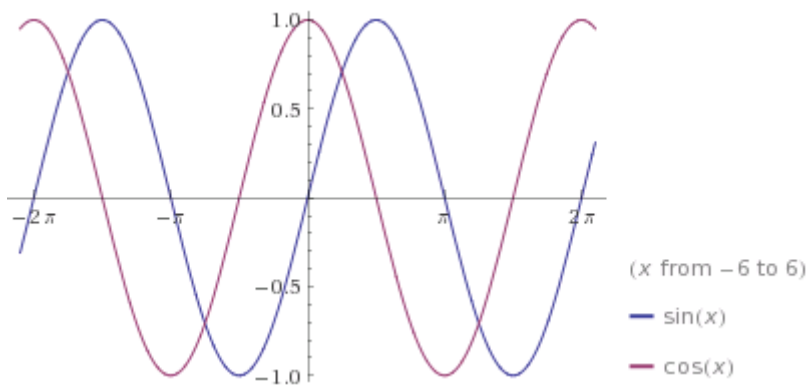
sin(x) is an odd function and cos(x) is an even function.
a) Define f(x) as a modification of sin(x) so that f is even.
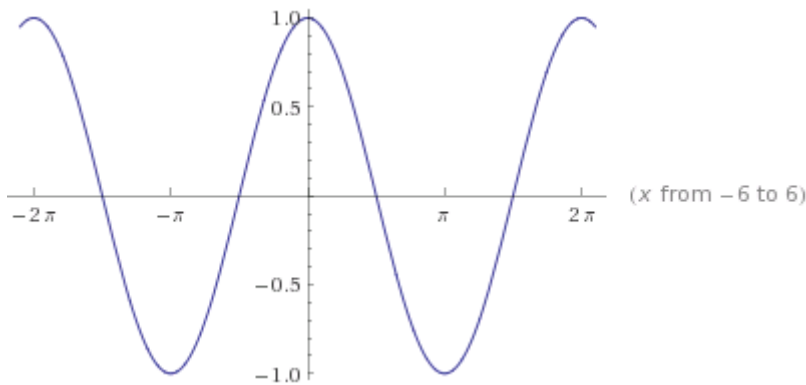b) Define g(x) as a modification of cos(x) so that g is odd.

Solution:

Let's build graph both (sin(x) and cos(x)) functions on one plot:

(a)

On the plot above we see that sin(x) shifted by $\frac{\pi}{2}$ to the left, it will be superimposed on the cosine graph.
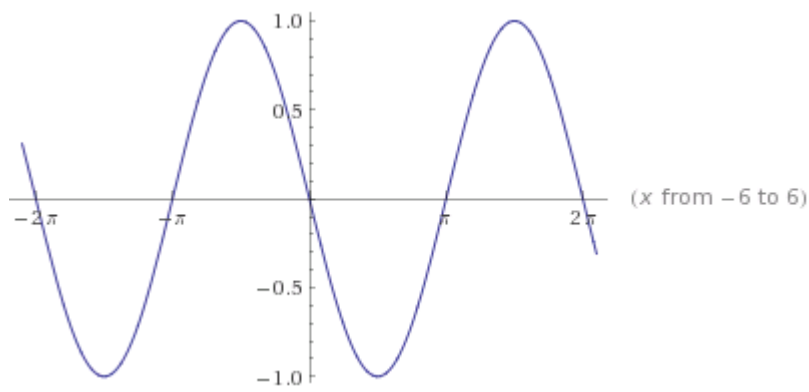

So $\sin(x + \frac{\pi}{2})$ = cos(x) and cos(x) is an even function. We get:

$$f(x) = \sin(x + \frac{\pi}{2})$$

(b)

On the plot above we see that cos(x) shifted by $\frac{\pi}{2}$ to the left, looks so:

(x from −6 to 6)

And

$$-\cos\left(x + \frac{\pi}{2}\right) = \cos\left(-\left(x + \frac{\pi}{2}\right)\right)$$

So $\cos\left(x + \frac{\pi}{2}\right)$ is an odd function. We get:

$$g(x) = \cos\left(x + \frac{\pi}{2}\right)$$

Answer: (a) $f(x) = \sin\left(x + \frac{\pi}{2}\right)$ and (b) $g(x) = \cos\left(x + \frac{\pi}{2}\right)$.