Fix a commutative ring S with a nilradical J that is not nilpotent, and consider $R_0 = \begin{pmatrix} J & S \\ J & J \end{pmatrix} \subseteq M_2(S)$. This R_0 is a "subring" of $M_2(S)$, except for the fact that it may not possess an identity element.
Then R_0 is a "nil rng" (i.e. every element of R_0 is nilpotent).
We adjoin an identity to R_0 to form a ring $R := R_0 \oplus Z$. Previous claim clearly implies that $Nil^*R = R_0$. Therefore, $Nil^*R \subseteq R0$. Let $N = N1(R) \subseteq Nil^*R$ be the sum of all nilpotent ideals of R. Then the matrix $\beta = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ does not belong to N. Moreover $\beta \in Nil^*R$,which implies that N is strictly contained in Nil^*R.