Assume now ideals in R satisfy DCC. Then $N^n = N^{n+1}$ for some n. We finish by showing that $M := N^n$ is zero. To see this, assume instead $M <> 0$. Then there exist ideals $A \subseteq M$ with $MAM <> 0$ (for instance $A = M$). Among such ideals A, choose a B that is minimal. Then $MBM <> 0$, so $MbM <> 0$ for some $b \in B$. Since $MbM \subseteq B$ and $M(MbM)M = MbM <> 0$, we must have $MbM = B$. In particular, there exists an equation

$b = \sum_{i=1}^{r} m_i b m_i'$, where $mi,m\_i \in M$.

Now consider the ideal $J \subseteq N$ generated by $\{m_i, m'_i : 1 \leq i \leq r\}$. Since N is a sum of nilpotent ideals, J lies in a finite sum of nilpotent ideals, so $J^k = 0$ for some k. Since $b \in JbJ$, we now have by repeated substitution $b \in J^k b J^k = 0$, a contradiction.