A boat is headed at 115° and is traveling downstream at 28 mph. The stream is flowing S38°E at 11 mph. In what direction is the boat traveling? At what rate can it travel in still water?

Solution:

$$S38°E = 180° - 38° = 142°$$

$\overrightarrow{v_1}$ – vector of the boat velocity in still water;

$\overrightarrow{v_2}$ – vector of the stream velocity;

$$|\overrightarrow{v_2}| = 11\ mph$$

$\vec{v}$ – the resultant velocity of the boat,

$$|\vec{v}| = 28\ mph$$

α – the angle between the headed direction of boat and the direction of stream,

β – the angle between the headed direction of boat and the direction the boat is traveling.

$$\alpha = 142° - 115° = 27°$$

$$\alpha = \beta + \gamma$$

Use the sine law:

$$\frac{|\vec{v}|}{\sin(180° - \alpha)} = \frac{|\overrightarrow{v_1}|}{sin\beta} = \frac{|\overrightarrow{v_2}|}{sin\gamma}$$

$$\gamma = arcsin\left(\frac{\overrightarrow{v_2} \cdot \sin(180° - \alpha)}{\vec{v}}\right) = arcsin\left(\frac{11 \cdot sin153°}{28}\right) = 10{,}27°$$

The direction the boat is traveling is the direction the boat is headed plus γ

$$115° + 10{,}27° = 125{,}27°$$

$$\beta = 27° - 10{,}27° = 16{,}73°$$

$$|\overrightarrow{v_1}| = \frac{sin\beta \cdot |\overrightarrow{v_2}|}{sin\gamma} = 17{,}76\ mph$$

Answer:

The boat is traveling $125{,}27°$. It can travel at $17{,}76\ mph$ in still water.