Conditions

I have got 35 coordinates (x;y) which seem to form a function of the second degree. I want to know the basic expression for this function, even if I just have the coordinates. These are:
1,3 3,15 4,26 5,37 6,50 7,67 8,74 9,85 10,102 11,125 12,138 13,155 14,174 15,197 16,228 17,257 18,280 19,309 20,352 21,383 22,420 23,463 24,522 25,563 26,610 27,653 28,700 29,761 30,832 31,911 32,970 33,1037 34,1098 35,1181

Solution

We have to use one of polynomial interpolation methods.

Let's find our function in the following form:

$$\tilde{y} = A_0 + A_1x + A_2x^2 .$$

Construct a difference:

$$S_m = \sum_{i=1}^{n}\left[y(x_i) - y_i\right]^2 = \sum_{i=1}^{n}\left[\left(A_0 + A_1x_i + A_2x_i^2\right) - y_i\right]^2$$

Take the derivatives:

$$\frac{\partial S_m}{\partial A_0} = 2\sum_{i=1}^{n}\left[\left(A_0 + A_1x_i + A_2x_i^2\right) - y_i\right] = 0,$$

$$\frac{\partial S_m}{\partial A_1} = 2\sum_{i=1}^{n}\left[\left(A_0 + A_1x_i + A_2x_i^2\right) - y_i\right]x_i = 0,$$

$$\frac{\partial S_m}{\partial A_2} = 2\sum_{i=1}^{n}\left[\left(A_0 + A_1x_i + A_2x_i^2\right) - y_i\right]x_i^2 = 0,$$

Now we have a system of equations:

$$\begin{cases} nA_0 + A_1\sum_{i=1}^{n}x_i + A_2\sum_{i=1}^{n}x_i^2 - \sum_{i=1}^{n}y_i = 0, \\ A_0\sum_{i=1}^{n}x_i + A_1\sum_{i=1}^{n}x_i^2 + A_2\sum_{i=1}^{n}x_i^3 - \sum_{i=1}^{n}x_iy_i = 0, \\ A_0\sum_{i=1}^{n}x_i^2 + A_1\sum_{i=1}^{n}x_i^3 + A_2\sum_{i=1}^{n}x_i^4 - \sum_{i=1}^{n}x_i^2y_i = 0 \end{cases}$$

After normalization we have the final form to find the coefficients A0, A1, A2:

$$\begin{cases} A_2\sum_{i=1}^{n} x_i^4 + A_1\sum_{i=1}^{n} x_i^3 + A_0\sum_{i=1}^{n} x_i^2 + = \sum_{i=1}^{n} x_i^2 y_i \\ A_2\sum_{i=1}^{n} x_i^3 + A_1\sum_{i=1}^{n} x_i^2 + A_0\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i \\ A_2\sum_{i=1}^{n} x_i^2 + A_1\sum_{i=1}^{n} x_i + nA_0 = \sum_{i=1}^{n} y_i \end{cases}$$

For solving this system we can use, for example, MS Excel or Wolfram Mathematics. Calculate there all these sums of x and y and their products, then find A0, A1, A2:

	x	y	x^2	x^3	x^4	x*y	X^2*y
	1	3	1	1	1	3	3
	3	15	9	27	81	45	135
	4	26	16	64	256	104	416
	5	37	25	125	625	185	925
	6	50	36	216	1296	300	1800
	7	67	49	343	2401	469	3283
	8	74	64	512	4096	592	4736
	9	85	81	729	6561	765	6885
	10	102	100	1000	10000	1020	10200
	11	125	121	1331	14641	1375	15125
	12	138	144	1728	20736	1656	19872
	13	155	169	2197	28561	2015	26195
	14	174	196	2744	38416	2436	34104
	15	197	225	3375	50625	2955	44325
	16	228	256	4096	65536	3648	58368
	17	257	289	4913	83521	4369	74273
	18	280	324	5832	104976	5040	90720
	19	309	361	6859	130321	5871	111549
	20	352	400	8000	160000	7040	140800
	21	383	441	9261	194481	8043	168903
	22	420	484	10648	234256	9240	203280
	23	463	529	12167	279841	10649	244927
	24	522	576	13824	331776	12528	300672
	25	563	625	15625	390625	14075	351875
	26	610	676	17576	456976	15860	412360
	27	653	729	19683	531441	17631	476037
	28	700	784	21952	614656	19600	548800
	29	761	841	24389	707281	22069	640001
	30	832	900	27000	810000	24960	748800
	31	911	961	29791	923521	28241	875471
	32	970	1024	32768	1048576	31040	993280
	33	1037	1089	35937	1185921	34221	1129293
	34	1098	1156	39304	1336336	37332	1269288
	35	1181	1225	42875	1500625	41335	1446725

Sum x,y	628	13778					
Sum x^2			14906				
Sum x^3				396892			
Sum x^4					11268962		
Sum x*y						366712	
Sum x^2*y							10453426

Now, using the Wolfram Mathematics, solve this system:

Input interpretation:

solve

$$14\,906\,x + 396\,892\,y + 11\,268\,962\,z = 10\,453\,426$$

$$628\,x + 14\,906\,y + 396\,892\,z = 366\,712$$

$$35\,x + 628\,y + 14\,906\,z = 13\,778$$

Result: More digits Step-by-step solution

$$x = \frac{1\,273\,197\,544}{43\,460\,607} \approx 29.295 \text{ and}$$

$$y = -\frac{1\,467\,718\,978}{304\,224\,249} \approx -4.8245 \text{ and } z = \frac{322\,111\,621}{304\,224\,249} \approx 1.0588$$

Here x is A0, y is A1 and z is A2. So, our function approximation is:

$$\tilde{y} = 29.295 - 4.8245x + 1.0588x^2$$