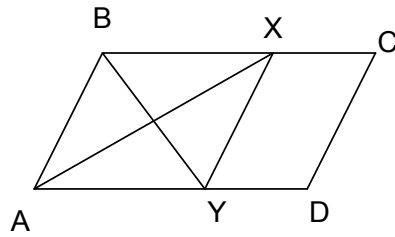


ABCD is a parallelogram. The bisectors of $\angle A$ and $\angle B$ meet BC and AD (produced if necessary) at X and Y respectively.
 Prove $XY = CD$



If ABCD is a parallelogram, then $AB \parallel CD$, $BC \parallel AD$, $AB = CD$ and $AD = BC$.

In $\triangle ABX$ $\angle BAX = \angle BXA$ because

$$\angle BAX = \angle XAY \text{ (AX is the bisector } \angle A \text{ – given)}$$

$$\angle BXA = \angle XAY \text{ (since } BC \parallel AD \text{ – given), so } \angle BAX = \angle BXA$$

Hence $\triangle ABX$ is an isosceles triangle, so $AB = BX$

In $\triangle BAY$ $\angle ABY = \angle AYB$ because

$$\angle ABY = \angle YBX \text{ (BY is the bisector } \angle B \text{ – given)}$$

$$\angle AYB = \angle YBX \text{ (since } BC \parallel AD \text{ – given), so } \angle ABY = \angle AYB$$

Hence $\triangle BAY$ is an isosceles triangle, so $AB = AY$

Hence $BX = AY$

In ABXY $BX = AY$ and $BX \parallel AY$ – given, so ABXY is a parallelogram and so $AB = XY$, but $AB = CD$ – given, so $XY = CD$.