If tan(A - 2B) = cot(2A - B) and tan(A + 2B) = cot(2A + B) , show both A and B are multiple of (π/6) and if A is odd multiples then B is even multiples.

Solution:

$$\cot\alpha = \tan\left(\frac{\pi}{2} - \alpha\right)$$

Hence the first is

$$\tan(A - 2B) = \tan\left(\frac{\pi}{2} - (2A - B)\right)$$

$(A - 2B) = \frac{\pi}{2} - (2A - B) + k_1\pi$, where k_1 is an integer

And the second is

$$\tan(A + 2B) = \tan\left(\frac{\pi}{2} - (2A + B)\right)$$

$(A + 2B) = \frac{\pi}{2} - (2A + B) + k_2\pi$, where k_2 is an integer

Solving the system of equations

$$(A - 2B) = \frac{\pi}{2} - (2A - B) + k_1\pi$$

$$(A + 2B) = \frac{\pi}{2} - (2A + B) + k_2\pi$$

From here

$$3A - 3B = \frac{\pi}{2} + k_1\pi$$

$$3A + 3B = \frac{\pi}{2} + k_2\pi$$

And finally adding or subtracting

$$6A = \pi + (k_1 + k_2)\pi$$

$$A = (k_1 + k_2 + 1)\frac{\pi}{6}$$

$$6B = (k_2 - k_1)\pi$$

$$B = (k_2 - k_1)\frac{\pi}{6}$$

Hence A and B are multiple of $\frac{\pi}{6}$. When k_1 and k_2 are both odd or even A is odd multiples then B is even multiples, and otherwise A is even multiples then B is odd multiples.