Solve the following set of simultaneous equations:

$$\begin{cases} \left(\frac{x}{5}\right) + \left(\frac{2y}{3}\right) - \left(\frac{49}{15}\right) = 0 \\ \left(\frac{3}{7}x\right) - \left(\frac{y}{2}\right) = -\frac{5}{7} \end{cases}$$

Using:

a) Substitution

b) Elimination

Solution:

a) Substitution

A set of simultaneous equations in n unknowns is linearly independent if none of the equations may be written as a sum of multiples of the others. Solution of such a set of equations can be done by substitution, addition of multiples of equations, or combinations of these. The method of solving "by substitution" works by solving one of the equations (you choose which one) for one of the variables (you choose which one), and then plugging this back into the other equation, "substituting" for the chosen variable and solving for the other.

In our case we have to solve one of the equations for one of the variables, and plug this into the other equation. It does not matter which equation or which variable we pick. To simplify the decision to get rid of the fraction for the first equation is multiplied by 15, the second at 14, resulting in a whole number of variables

$$\begin{cases} \left(\frac{x}{5}\right) + \left(\frac{2y}{3}\right) - \left(\frac{49}{15}\right) = 0 & \times 15 \\ \left(\frac{3}{7}x\right) - \left(\frac{y}{2}\right) = -\frac{5}{7} & \times 14 \end{cases}$$

$$\begin{cases} 3x + 10y = 49 \\ 6x - 7y = -10 \end{cases}$$

In this case we can see that it would probably be simplest to solve the first equation for "x =".I

$$\begin{cases} 3x = 49 - 10y \\ x = \frac{(49 - 10y)}{3} \end{cases}$$

Substitute x into second equation:

$$\begin{cases} 6(\frac{49-10y}{3}) - 7y = -10 \\ x = \frac{(49-10y)}{3} \end{cases}$$

We will plug this in ("substitute it") for "x" in the second equation, and solve for y:

$$\begin{cases} 6(\frac{49-10y}{3}) - 7y = -10 \\ 98 - 20y - 7y = -10 \end{cases}$$

$$\begin{cases} -27y = -10 - 98 \\ -27y = -108 \end{cases}$$

$$y = 4$$

Now we can plug this y-value back into either equation, and solve for x.

$$\begin{cases} x = \frac{(49-10y)}{3} \\ y = 4 \end{cases}$$

$$x = \frac{(49 - 10 \cdot 4)}{3} = \frac{49 - 40}{3} = \frac{9}{3} = 3$$

The solutions of the following set of simultaneous equations are:

$$x = 3$$

$$y = 4$$

Solve the following set of simultaneous equations:

b) Elimination

The elimination method of solving systems of equations is also called the addition method. To solve a system of equations by elimination we transform the system such that one variable "cancels out". As in previous method to simplify the decision to get rid of the fraction for the first equation is multiplied by 15, the second at 14, resulting in a whole number of variables

$$\begin{cases} 3x + 10y = 49 & \text{Multiply by } -2 \\ 6x - 7y = -10 \end{cases}$$

Look at the x - coefficients. Multiply the first equation by -2, to set up the x-coefficients to cancel.

$$\begin{cases} -6x - 20y = -98 \\ 6x - 7y = -10 \end{cases}$$

Next step we add equations:

$$\begin{cases} -6x - 20y = -98 \\ 6x - 7y = -10 \end{cases}$$

$$-27y = -108$$

$$y = 4$$

The same operation we can repeat with y. In this example, we will multiply the first row by 7 and the second row by 10; then we will add down as before.

$$\begin{cases} 3x + 10y = 49 & \text{Multiply by } 7 \\ 6x - 7y = -10 & \text{Multiply by } 10 \end{cases}$$

Next step we add equations:

$$\begin{cases} 21x + 70y = 343 \\ 60x - 70y = -100 \end{cases}$$

$$81x = 243$$

$$x = 3$$

We can check that these values are correct by substituting the values into equation:

$$\begin{cases} \left(\frac{3}{5}\right) + \left(\frac{2\cdot 4}{3}\right) - \left(\frac{49}{15}\right) = 0 \\ \left(\frac{3}{7}\cdot 3\right) - \left(\frac{4}{2}\right) = -\frac{5}{7} \end{cases}$$

Simplify the equations:

$$\begin{cases} \left(\frac{3}{5}\right) + \left(\frac{8}{3}\right) - \left(\frac{49}{15}\right) = 0 \\ \left(\frac{9}{7}\right) - \left(\frac{4}{2}\right) = -\frac{5}{7} \end{cases} \Rightarrow \begin{cases} \frac{9+40-49}{15} = 0 \\ \frac{18-28}{14} = -\frac{5}{7} \end{cases} \Rightarrow \begin{cases} 0 = 0 \\ -\frac{10}{14} = -\frac{5}{7} \end{cases} \qquad -\frac{5}{7} = -\frac{5}{7}$$

Answer: $x = 3\ y = 4$