

All statement implies from formula  $\det A = \sum (-1)^{[i_1, i_2, \dots, i_n]} a_{i_1 1} a_{i_2 2} \dots a_{i_n n}$ .

- 1) if we substitute  $a_{i_k k}$  by  $ta_{i_k k}$ , (row is multiplied by t) then we can bring t out of sum, and we are done.
- 2) as any transposition changes sign of permutation, then  $[i_1, i_2, \dots, i_n]$  will be changed from odd to even and vice versa in every summand, so sign of  $\det A$  will be opposite.
- 3) if one of rows is equal to another one, then their permutation change sign of  $\det A$ , and in general  $\det A$  will be the same. It can be iff  $\det A=0$ . So, if we have

$$\sum (-1)^{[i_1, i_2, \dots, i_n]} a_{i_1 1} a_{i_2 2} \dots (a_{i_k k} + ta_{i_l l}) \dots a_{i_n n} = \sum (-1)^{[i_1, i_2, \dots, i_n]} a_{i_1 1} a_{i_2 2} \dots (a_{i_k k}) \dots a_{i_n n} + t \sum (-1)^{[i_1, i_2, \dots, i_n]} a_{i_1 1} a_{i_2 2} \dots (a_{i_l l}) \dots a_{i_n n}$$

then in first summand all i's are distinct, and in second l is being used twice, so second summand is zero.