Question 1. *Why do we impose the requirement that the eigenvector be nonzero when we do not place this requirement on the eigenvalue.*

Solution. Each eigenvector should have a uniquely defined eigenvalue. Since $A0 = 0$ for any operator A, then we may write that $A0 = \lambda 0$ for any scalar λ. So, any number can be seen as an eigenvalue of the zero vector, if we allow 0 to be an eigenvector. To avoid this ambiguity, an eigenvector is assumed to be nonzero. □