Question 1. *Prove that $0v = 0$ for all $v \in V$.*

Solution. By distributivity of scalar multiplication with respect to field addition (axiom 6) we have

$$0v = (0 + 0)v = 0v + 0v.$$

By axiom 4 there exists an additive inverse $-0v$ of $0v$, such that $0v + (-0v)$ is the zero vector $0 \in V$. Then adding $-0v$ to both sides of the above equality by associativity of addition (axiom 1) and definition of the zero vector we get:

$$0 = (0v + 0v) + (-0v) = 0v + (0v + (-0v)) = 0v + 0 = 0v.$$

Thus, $0v = 0$ as desired. $\qquad\square$