

Task:

Verify the identities.

$$\sin^4 x - \cos^4 x = 2\sin^2 x - 1.$$

Solution:

Let's transform the left part of the expression:

We use the relationship $x^2 - y^2 = (x + y)(x - y)$ [difference between two squares]:

$$\sin^4 x - \cos^4 x = (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) =$$

Using the basic relationship between the sine and the cosine $\sin^2 x + \cos^2 x = 1$, we get:

$$= (1) \cdot (\sin^2 x - \cos^2 x) = \sin^2 x - \cos^2 x = \sin^2 x - \cos^2 x + 1 - 1$$

We add and subtract 1(one):

$$= \sin^2 x - \cos^2 x + 1 - 1$$

Using the basic relationship between the sine and the cosine, we have:

$$= \sin^2 x - \cos^2 x + \sin^2 x + \cos^2 x - 1 =$$

And simplified:

$$= 2 \sin^2 x - 1$$

And this is the same as the right part of the expression.

Solution: expressions are identical.