Question

AC is a diameter of circle centered at O and $m\angle BOC=54$. Find $m\angle BAO$.

Solution
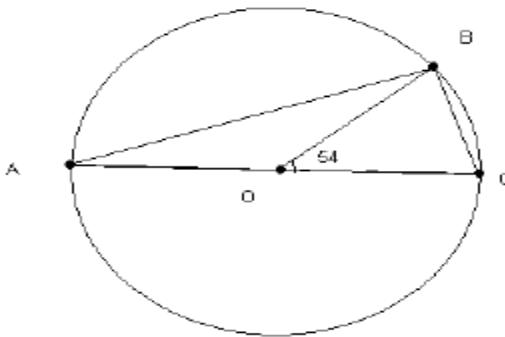
Given

$$m\angle BOC = 54^{\circ}$$

calculate

$$m\angle BOA = m\angle COA - m\angle COB = 180^{\circ} - 54^{\circ} = 126^{\circ}.$$

AO is a radius, BO is a radius too. Therefore, the triangle ΔABO is isosceles.

That's why

$$m\angle BAO = m\angle ABO.$$

The sum of the angles of a triangle is 180°, that is,

$$m\angle BAO + m\angle ABO + m\angle BOA = 180^{\circ},$$

$$m\angle BAO + m\angle ABO + 126^{\circ} = 180^{\circ},$$

$$m\angle BAO + m\angle ABO = 180^{\circ} - 126^{\circ},$$

$$m\angle BAO + m\angle ABO = 54^{\circ}.$$

Thus, we get the system of equations $\begin{cases} m\angle BAO + m\angle ABO = 54^{\circ}, \\ m\angle BAO = m\angle ABO. \end{cases}$

Consider

$$m\angle BAO + m\angle ABO = m\angle BAO + m\angle BAO = 2m\angle BAO = 54^{\circ},$$

that is,

$$2m\angle BAO = 54^{\circ},$$

dividing by 2 obtain

$$m\angle BAO = 27^{\circ}.$$

Answer: $m\angle BAO = 27^{\circ}$.