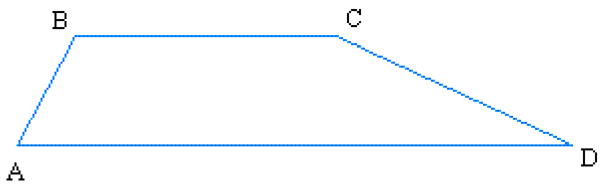
prove that area of a trapezium=1/2(a+b)h

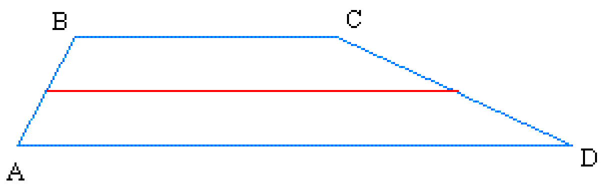
Proof of the area of a trapezoid

A first good way to start off with the proof of the area of a trapezoid is to draw a trapezoid and turn the trapezoid into a rectangle.

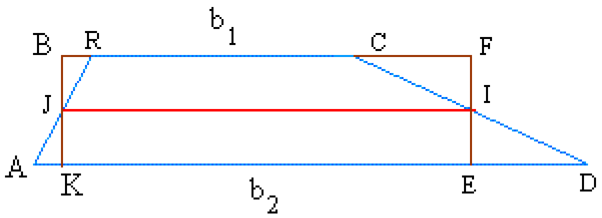


Look at the trapezoid ABCD above. How would you turn this into a rectangle?

Draw the average base (shown in red) which connects the midpoint of the two sides that are not parallel



Then, make 4 triangles as shown below:



Let's call the two parallel sides in blue (the bases) b_1 and b_2

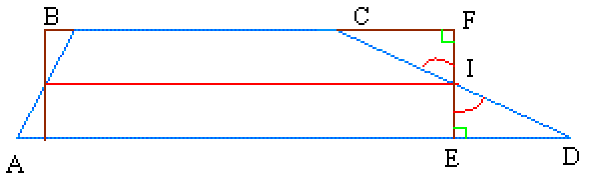
Since triangles EDI and CFI are congruent or equal and triangles KAJ and RBJ are equal, you could make a rectangle by rotating triangles EDI around point I, 180 degrees counterclockwise.

And by rotating triangle KAJ clockwise, but still 180 degrees around point J.

Because you could make a rectangle with the trapezoid, both figures have the same area

The reason that triangle EDI is equal to triangle IFC is because we can find two angles inside the triangles that are the same. If two angles are the same, then the third or last angle must be the same

The angles that are the same are shown below. They are in red and green. The angles in green are right angles. The angles in red are vertical angles

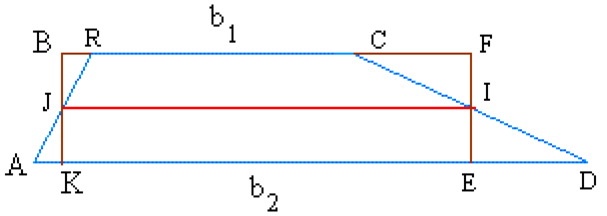


This is important because if these two triangles are not congruent or the same, we cannot make the rectangle with the trapezoid by rotating triangle EDI. It would not fit properly

Again, this same argument applies for the two triangles on the left

Therefore, if we can find the area of the rectangle, the trapezoid will have the same area

Let us find the area of the rectangle.We will need the following figure again:



First, make these important observations:

b_1 = RC

BF = BR + b_1 + CF

b_2 = AD

KE = AD − AK − ED, so KE = b_2 − AK − ED

AK = BR and ED = CF

Notice also that you can find the length of the line in red (the average base) by taking the average of length BF and length KE

$$\text{Therefore,} \quad JI = \frac{BF + KE}{2}$$

$$= \frac{BR + b_1 + CF + b_2 - AK - ED}{2}$$

$$= \frac{BR + b_1 + CF + b_2 - BR - CF}{2}$$

$$= \frac{\cancel{BR} + b_1 + \cancel{CF} + b_2 - \cancel{BR} - \cancel{CF}}{2}$$

$$= \frac{b_1 + b_2}{2}$$

Since the length of the line in red is the same as the base of the rectangle, we can just times that by the hegiht to get the area of the trapezoid

Finally, we get :

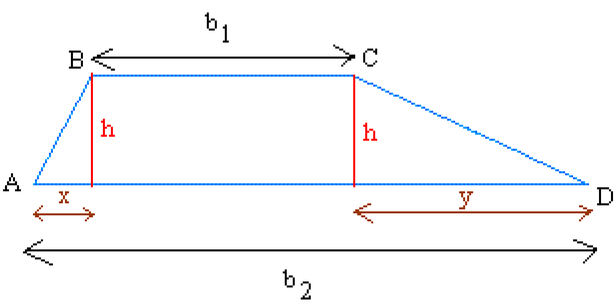
$$\text{Area} = \frac{b_1 + b_2}{2} \bullet h$$

An alternative proof of the area of a trapezoid could be done this way.

Start with the same trapezoid. Draw heights from From vertex B and C. This will break the trapezoid down into 3 shapes: 2 triangles and a rectangle.

Label the base of the small triangle x and the base of the bigger triangle y

Label the small base of the trapezoid b_1 and b_2



b1 = b2 − (x + y), so x + y = b2 − b1

The area of the rectangle is b1 × h, but the area of the triangles with base x and y are :

$$\frac{x \text{ times } h}{2} \quad \text{and} \quad \frac{y \text{ times } h}{2}$$

To get the total area, just add these areas together:

$$\text{Total area} = \frac{x \bullet h}{2} + \frac{y \bullet h}{2} + b_1 \bullet h$$

$$= \frac{x \bullet h}{2} + \frac{y \bullet h}{2} + \frac{2 \bullet b_1 \bullet h}{2}$$

$$= \frac{x \bullet h + y \bullet h + 2 \bullet b_1 \bullet h}{2}$$

$$= \frac{(x + y + 2 \bullet b_1) \bullet h}{2}$$

$$= \frac{(b_2 - b_1 + 2 \bullet b_1) \bullet h}{2} \qquad x + y = b_2 - b_1$$

$$= \frac{(b_2 + b_1) \bullet h}{2}$$

The proof of the area of a trapezoid is complete.