Question 1. *How do I write the expression that would take a number x and square it, and then square the result of the first operation and continue on for n generations?*

Solution. After the first operation we shall obtain x^2. Applying the same operation twice we get
$$(x^2)^2 = x^{2^2}.$$
We shall prove by induction that after n applications the result will be x^{2^n}.

The base of induction: for $n = 1$ the result is $x^2 = x^{2^1}$ as was mentioned above. The inductive step: suppose the formula is true for n, i.e. the result is x^{2^n}. We need to show that it is true for $n + 1$. Indeed,

$$\left(x^{2^n}\right)^2 = x^{2^n \cdot 2} = x^{2^{n+1}}.$$

Answer: The result after n operations is x^{2^n}.

$\square$