

QUESTION:

$\nabla^p(a_0 + a_1 t + \dots + a_n t^p) = p! * a_n$ prove left hand side.. and note that a_0, a_1, \dots, a_n is in the subscript of a and $\wedge p$ means power and $*$ means multiply.

SOLUTION:

$$\begin{aligned}\nabla^p(a_0 + a_1 t + a_2 t^2 + \dots + a_n t^p) &= \nabla^p(a_0) + \nabla^p(a_1 t) + \dots + \nabla^p(a_n t^p) = a_n \nabla^p(t^p) \\ (\text{since } \nabla^p(a_0) &= 0, \quad \nabla^p(a_1 t) = 0, \quad \dots, \quad \nabla^p(a_{n-1} t^{p-1}) = 0) \text{ . Hence} \\ a_n \nabla^p(t^p) &= a_n \cdot p \cdot \nabla^{p-1}(t^{p-1}) = a_n \cdot p \cdot (p-1) \nabla^{p-2}(t^{p-2}) = \dots = a_n \cdot p(p-1)(p-2)(p-3)\dots1 = a_n p!\end{aligned}$$