Conditions

Consider the equation yu_x - xu_y = 0, (y>0). Check for each of the following initial conditions whether the problem is solvable. If it is solvable, find a solution. If it is not, explain why:

a) u(x,0) = x^2
b) u(x,0) = x
c) u(x,0) = x, x>0

Solution

$$y\frac{du}{dx} - x\frac{du}{dy} = 0$$

$$\frac{dx}{y} = \frac{dy}{x} = \frac{du}{0}$$

$$\varphi_1(x,y,u) = u$$

$$\frac{dx}{y} = \frac{dy}{x}$$

$$x^2 - y^2 = c$$

$$\Phi(u, x^2 - y^2) = 0$$

$$u = f(x^2 - y^2)$$

Where f has a derivative at some interval.

So we can see now, that the problem is solvable for 1st condition.