Conditions

prove pythagorous theoram

Solution

In mathematics, the Pythagorean theorem — or Pythagoras' theorem — is a relation in Euclidean geometry among the three sides of a right triangle (right-angled triangle). In terms of areas, it states:

In any right-angled triangle, the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares whose sides are the two legs (the two sides that meet at a right angle).

The theorem can be written as an equation relating the lengths of the sides a, b and c, often called the Pythagorean equation:

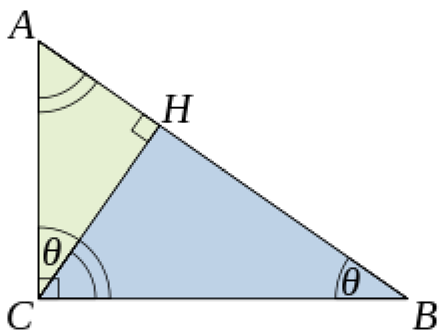
$$a^2 + b^2 = c^2$$

where c represents the length of the hypotenuse, and a and b represent the lengths of the other two sides.

There are many proofs of this theorem. We will prove it by using one of the most simple proofs – using similar triangles.

This proof is based on the proportionality of the sides of two similar triangles, that is, upon the fact that the ratio of any two corresponding sides of similar triangles is the same regardless of the size of the triangles.

Let ABC represent a right triangle, with the right angle located at C, as shown on the figure.



We draw the altitude from point C, and call H its intersection with the side AB. Point H divides the length of the hypotenuse c into parts d and e. The new triangle ACH is similar to triangle ABC, because they both have a right angle (by definition of the altitude), and they share the angle at A, meaning that the third angle will be the same in both triangles as well, marked as θ in the figure. By a similar reasoning, the triangle CBH is also similar to ABC. The proof of similarity of the triangles requires the Triangle postulate: the sum of the angles in a triangle is two right angles, and is equivalent to the parallel postulate. Similarity of the triangles leads to the equality of ratios of corresponding sides:

$$\frac{BC}{AB} = \frac{BH}{BC} \text{ and } \frac{AC}{AB} = \frac{AH}{AC}.$$

The first result equates the cosine of each angle θ and the second result equates the sines.

These ratios can be written as:

$$BC^2 = AB \times BH \text{ and } AC^2 = AB \times AH.$$

Summing these two equalities, we obtain

$$BC^2 + AC^2 = AB \times BH + AB \times AH = AB \times (AH + BH) = AB^2,$$

which, tidying up, is the Pythagorean theorem:

$$BC^2 + AC^2 = AB^2 \ .$$

Q.E.D.

The role of this proof in history is the subject of much speculation. The underlying question is why Euclid did not use this proof, but invented another. One conjecture is that the proof by similar triangles involved a theory of proportions, a topic not discussed until later in the Elements, and that the theory of proportions needed further development at that time.