

QUESTION:

The heights of 10 year old boys and girls closely follow a normal distribution with the mean of 51 inches and standard deviation of 2 inches.

- what percentage of 10 year olds are shorter than 50 inches?
- What percentage of 10 yr olds are between 48 and 54?

SOLUTION:

So, normal distribution can be written as:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \text{ where}$$

$$\mu = 51$$

$$\sigma = 2$$

a) The percentage of 10 year olds, older than 50 inches can be found with the help of cumulative integral function of normal distribution:

$$\begin{aligned} \Phi(50) - \Phi(0) &= \int_{-\infty}^{50} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx - \int_{-\infty}^0 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \\ &= \frac{1}{2} \left(1 + \operatorname{erf}\left(\frac{50-51}{2\sqrt{2}}\right) \right) - \frac{1}{2} \left(1 + \operatorname{erf}\left(\frac{0-51}{2\sqrt{2}}\right) \right) = \frac{1}{2} \left(\operatorname{erf}\left(-\frac{\sqrt{2}}{4}\right) - \operatorname{erf}\left(-\frac{51\sqrt{2}}{4}\right) \right) = \\ &= \frac{1}{2} \left(\operatorname{erf}\left(\frac{51\sqrt{2}}{4}\right) - \operatorname{erf}\left(\frac{\sqrt{2}}{4}\right) \right) \end{aligned}$$

$\operatorname{erf}(x)$ is the error function and this function is odd function ($\operatorname{erf}(-x) = -\operatorname{erf}(x)$). So,

$$\Phi(50) - \Phi(0) \approx 0.309, \text{ or } 30.9\%$$

b) The percentage of 10 yr olds are between 48 and 54 can be found with the help of cumulative integral function of normal distribution:

$$\begin{aligned} \Phi(54) - \Phi(48) &= \int_{-\infty}^{54} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx - \int_{-\infty}^{48} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \\ &= \frac{1}{2} \left(1 + \operatorname{erf}\left(\frac{54-51}{2\sqrt{2}}\right) \right) - \frac{1}{2} \left(1 + \operatorname{erf}\left(\frac{48-51}{2\sqrt{2}}\right) \right) = \frac{1}{2} \left(\operatorname{erf}\left(\frac{3\sqrt{2}}{4}\right) - \operatorname{erf}\left(-\frac{3\sqrt{2}}{4}\right) \right) = \\ &= \operatorname{erf}\left(\frac{3\sqrt{2}}{4}\right) \end{aligned}$$

$$\Phi(54) - \Phi(48) \approx 0.866 \text{ or } 86.6\%$$

ANSWER:

a) 0.309 or 30.9%

b) 0.866 or 86.6%