Conditions

The number of values of k for which the system of equations x+y=2 , kx+y=4 , x+ky=5 has atleast one solution?

Solution

We must construct a system of 3 equations, where there will be 3 variables: x, y and k:

$$\begin{cases} x+y=2 \\ kx+y=4 \\ x+ky=5 \end{cases}$$

$$x = 2 - y$$

3rd equation minus 1st give us:

$$(k-1)y = 3$$

$$y = \frac{3}{k-1}$$

Then, from the second equation:

$$k\left(2 - \frac{3}{k-1}\right) + \frac{3}{k-1} = 4$$

It's obvious, that k=1 couldn't be a solution (because all 3 equations would have equal left sides, but the right sides wouldn't be equal). So, let's multiply on (k-1):

$$2k(k-1) - 3k + 3 = 4(k-1)$$

$$2k^2 - 2k - 3k + 3 - 4k + 4 = 0$$

$$2k^2 - 9k + 7 = 0$$

$$D = 81 - 4 \cdot 2 \cdot 7 = 81 - 56 = 25 > 0$$

On this point we can already answer our question – as the discriminant is positive – there are 2 different solutions for k, that's why the number of values of k, for which the system of equations has at least one solution is **TWO VALUES**

To make sure it, we can find 2 values of k:

$$k_{1,2} = \frac{9 \pm 5}{4} = \begin{bmatrix} k = \frac{7}{2} \\ k = 1 \end{bmatrix}$$

After that we will have 2 pairs of x and y, for each k, which could be found by substitution k-value in these equations:

$$x = 2 - y$$

$$y = \frac{3}{k - 1}$$