

Can you please help me evaluate this indefinite integral?

$$\int [1/(x\sqrt{(x^2)-4})]dx$$

$$I = \int \frac{dx}{x\sqrt{x^2-4}}$$

Let $x = \frac{2}{\cos t}$, then

$$t = \cos^{-1} \left| \frac{2}{x} \right|$$

$$\sqrt{x^2-4} = \sqrt{\frac{4}{\cos^2 t} - 4} = \sqrt{\frac{4(1-\cos^2 t)}{\cos^2 t}} = \sqrt{\frac{4\sin^2 t}{\cos^2 t}} = \frac{2\sin t}{\cos t}$$

and

$$dx = \frac{2\sin t}{\cos^2 t} dt$$

So

$$I = \int \frac{\frac{2\sin t}{\cos^2 t} dt}{\frac{2}{\cos t} \times \frac{2\sin t}{\cos t}} = \int \frac{1}{2} dt = \frac{1}{2}t + C = \frac{1}{2}\cos^{-1} \left| \frac{2}{x} \right| + C$$