In preparation for the computation of the character table of G, we first note that G has *five* conjugacy classes, represented by $1, a, a^3, b, b^2$. (This is an easy group-theoretic computation, which we omit.) Thus, we expect to have *five* irreducible complex representations. Obviously, $[G,G] = \langle a \rangle$, so $G/[G,G] \sim \langle b \rangle$. This shows that there are three 1-dimensional representations $\chi_i : G \to \mathrm{C}^*$, which are trivial on $\langle a \rangle$, with $\chi_1(b) = 1$, $\chi_2(b) = \omega$, and $\chi_3(b) = \omega^2$, where ω is a primitive cubic root of unity. Next, we construct a 3-dimensional C-representation $D : G \to \mathrm{GL}_3(\mathrm{C})$ by taking

$$D(a) = \begin{pmatrix} \zeta & & \\ & \zeta^2 & \\ & & \zeta^4 \end{pmatrix}, \text{ and } D(b) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

where ζ is a primitive 7th root of unity. (It is straightforward to check that the relations between a and b are respected by D.) If D is a reducible representation, it would have to “contain” a 1-dimensional representation. This is easily checked to be not the case. Thus, D is irreducible, and we get another irreducible 3-dimensional C-representation D' by taking

$$D'(a) = \overline{D(a)} = \begin{pmatrix} \zeta^6 & & \\ & \zeta^5 & \\ & & \zeta^3 \end{pmatrix}, \text{ and } D'(b) = \overline{D(b)} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

Note that we have $D \ncong D'$, since they have different characters, say χ_4 and χ_5. We have now computed all complex irreducible representations of G, arriving at the following character table:

	1	a	a^3	b	b^2
χ_1	1	1	1	1	1
χ_2	1	1	1	ω	ω
χ_3	1	1	1	ω	ω
χ_4	3	α	α	0	0
χ_5	3	α	α	0	0

(where $\alpha = \zeta + \zeta^2 + \zeta^4$).

From the first column of this character table, we see that the Wedderburn decomposition of CG is:

$\mathrm{C}G \sim \mathrm{C} \times \mathrm{C} \times \mathrm{C} \times \mathrm{M}_3(\mathrm{C}) \times \mathrm{M}_3(\mathrm{C})$.