Let ζ be a primitive p - th root of unity, and let $G = \langle x \rangle$. The map $x \to \zeta$ extends to a surjective ring homomorphism $QG \to Q(\zeta)$. Therefore, one of the simple components of QG is $Q(\zeta)$. On the other hand, QG also has a simple component Q. Since $\dim_Q(Q \times Q(\zeta)) = 1+p-1 = p$, we must have $QG \sim Q \times Q(\zeta)$. For any commutative ring R, let $U_0(R)$ denote the group of units of finite order in R. Then $U_0(QG) \sim U_0(Q) \times U_0(Q(\zeta)) \sim \{\pm 1\} \times U_0(Q(\zeta))$. We finish by noting that $U_0(Q(\zeta))$ is the group of roots of unity in $Q(\zeta)$, which is just the group $\langle -\zeta \rangle = \langle \pm 1 \rangle \times \langle \zeta \rangle$