In the following, let $U_+$ denote the trivial representation, and $U_-$ the sign representation of $kG$. We shall construct four more indecomposable kG-modules: M,M' of dimension 3, and V, V' of dimension 2. Let M = $ke_1 \oplus ke_2 \oplus ke_3$, on which $G$ acts by permuting the $ei$'s, and let $V = M/k(e_1 + e_2 + e_3)$.

1) Let $V_0 = kv$, where $v = e_1 - e_2 \in V$. We check easily that $V_0$ is a $kG$ – submodule of $V$ isomorphic to $U_-$, with $V/V_0 \sim U_+$. Thus, the composition factors for $V$ are $\{U_+, U_-\}$. We claim that $V$ has no submodule $\sim U_+$. In fact, if 0 is not equal $u = ae_1 + be_2$ spans a line $\sim U+$, then $ae_1 + be_2 = (12)u = ae_2 + be_1$ and $ae_1 + be_2 = (123)u = ae_2 + be_3 = -be_1 + (a - b)e_2$ imply that $a = b = 0$, a contradiction. It follows that $V$ is indecomposable, and hence so is $V' := U_- \otimes V$ . Also, since $V'$ contains $U_- \otimes U_- \sim U_+$, we see that $V$ is not isomorphic to the $V'$.

(b2) Using the same type of calculations as above, we can verify the following two properties of $M = ke1 \oplus ke2 \oplus ke3$:

(A) *M has no kG-submodule* $\sim U_-$.

(B) *The only kG-submodule of M isomorphic to* $U_+$ *is* $k(e_1 + e_2 + e_3)$.

We can now show that $M$ is indecomposable. In fact, if otherwise, (A) implies that $M \sim U_+ \oplus N$ for some $N$. Since $M$ has composition factors $\{U_+, U_+, U_-\}$, $N$ has composition factors $\{U_+, U_-\}$. By (A) again, $N$ must contain a copy of $U_+$. But then $M$ contains a copy of $U_+ \oplus U_+$, which contradicts (B). This shows that $M$ is indecomposable, and hence so is $M' := U_- \otimes M$. Since $M \supseteq k(e1 + e2 + e3) \sim U_+$, $M'$ contains a copy of $U_- \otimes U_+ \sim U_-$. By (A) again, we have $M$ is not isomorphic to the $M'$. This completes the construction of the six indecomposable *kG*-modules.