Let U be an irreducible kG-module, and let T be the action of (123) on U. Since $(T - I)^3 = T^3 - I = 0$, T has an eigenvalue 1, so $U_0 = \{u \in U : Tu = u\}$ is nonzero. For $u \in U_0$, we have $(123)((12)u) = (12)(132)u = (12)(123)^2 u = (12)u$, so $(12)u \in U_0$ also. This shows that U_0 is a kG-submodule of U, so $U_0 = U$, and we may view U as a kG-module where $G = G/ \langle(123)\rangle = \langle(12)'\rangle$. Since (12) has order 2, we have $U = U_+ \oplus U_-$ where $U_+ = \{u \in U : (12)u = u\}$, and $U_- = \{u \in U : (12)u = -u\}$. Therefore, we have either $U = U_+$ (giving the trivial representation), or $U = U_-$ (giving the sign representation).

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