Question 1. *Let $A = \emptyset$.*

1. *Is it an equivalence relation?*

2. *Is it antisymmetric?*

Solution. 1. By definition, a relation A on X is an equivalence relation if and only if it is reflexive, i. e. for all $x \in X$:

$$(x, x) \in A,$$

symmetric, i. e. for all $x, y \in X$:

$$(x, y) \in A \Rightarrow (y, x) \in A$$

and transitive, i. e. for all $x, y, z \in X$:

$$(x, y), (y, z) \in A \Rightarrow (x, z) \in A.$$

So, if $X \neq \emptyset$, then $A = \emptyset$ cannot be an equivalence relation, because it is not reflexive. Now suppose $X = \emptyset$. Then $A = \emptyset = X \times X$ is, obviously, reflexive, symmetric and transitive, because all the premises of the implications above are always false (there are no elements in X) and hence the implications are true.

2. A relation A on X is antisymmetric if for all $x, y \in X$:

$$(x, y), (y, x) \in A \Rightarrow x = y.$$

Clearly, if $A = \emptyset$, then the premise of the implication is false, so the implication is true and thus A is antisymmetric.
Answer:

1. If $X = \emptyset$, then A is an equivalence relation on X, otherwise it is not an equivalence relation on X;

2. A is antisymmetric.

□