Conditions

The point P(0.25, 8) lies on the curve y=2/x . Let Q be the point (x, 2/x)

a.) Find the slope of the secant line PQ for the following values of x.

If x=0.35, the slope of PQ is
If x=0.26, the slope of PQ is
If x=0.15, the slope of PQ is
If x=0.24, the slope of PQ is

b.) Based on the above results, guess the slope of the tangent line to the curve at P(0.25, 8).

Solution

The concept of a slope is central to differential calculus. For non-linear functions, the rate of change varies along the curve. The derivative of the function at a point is the slope of the line tangent to the curve at the point, and is thus equal to the rate of change of the function at that point.

If we let Δx and Δy be the distances (along the x and y axes, respectively) between two points on a curve, then the slope given by the above definition,

$$m = \frac{\Delta y}{\Delta x}$$

is the slope of a secant line to the curve. For a line, the secant between any two points is the line itself, but this is not the case for any other type of curve.

a) So, let's calculate it for our exercise:

$$m = \frac{8 - \frac{2}{x}}{0.25 - x}$$

And for all x we can substitute its values to find the relative m:

For x=0.35:

$$m = \frac{8 - \frac{2}{0.35}}{0.25 - 0.35} = -\frac{160}{7}$$

For x=0.26:

$$m = \frac{8 - \frac{2}{0.26}}{0.25 - 0.26} = -\frac{400}{13}$$

For x=0.15:

$$m = \frac{8 - \frac{2}{0.15}}{0.25 - 0.15} = -\frac{160}{3}$$

For x=0.24:

$$m = \frac{8 - \frac{2}{0.24}}{0.25 - 0.24} = -\frac{100}{3}$$

b) Let’s find the slope of the tangent line at a point P

According to a consequence of mean value theorem, we have:

$$k = \frac{\frac{2}{0.26} - \frac{2}{0.24}}{0.26 - 0.24} = \frac{0.48 - 0.52}{0.26 \cdot 0.24 \cdot 0.02} = -\frac{1250}{39}$$