Task:

Prove that the system:

$$u_x = 2.999999x^2y + y$$

$$u_{\nu} = x^3 + x$$

has no solution at all.

Solution:

$$u_y = x^3 + x$$

$$u = x^3y + xy + f(x)$$

$$(x^3y + xy + f(x))_x = 2.999999x^2y + y$$

$$3x^2y + y + f'(x) = 2.999999x^2y + y$$

$$2.999999x^2y + y + (f'(x) + 0.000001x^2y) = 2.999999x^2y + y$$

$$f'(x) = 0$$

There is no analogy of $0.000001x^2y$ in the right side. There are no solutions