**Question 23819**Show that the rank of a matrix C = AB is never greater than the smaller of the rank of A and the rank of B. Can it ever be strictly less than the smaller of these two numbers?.

**Solution.** Since $Cx = A(Bx)$, then $\text{rank}(C) \leq \text{rank}(A)$, next it is known that rank of transpose matrix is equal to the rank of the original matrix, thus, $\text{rank}(C') = \text{rank}(C)$ and $C'x = B'(A'(x))$, thus $\text{rank}(C') \leq \text{rank}(B') = \text{rank}(B)$, thus $\text{rank}(C) \leq \text{rank}(A)$, $\text{rank}(C) \leq \text{rank}(B)$, thus $\text{rank}(C) \leq \min\{\text{rank}(A), \text{rank}(B)\}$.