Question 23818 Let u and v be two non-zero N-dimensional complex column vectors. Show that the rank of the $N \times N$ matrix $u\overline{v}'$ is one.
Solution. We know the following general inequality $\operatorname{rank}(u\overline{v}') \leq \min\{\operatorname{rank}(u), \operatorname{rank}(\overline{v}')\} = 1$, since u and $\overline{v}'$ are non-zero. Next $\operatorname{rank}(u\overline{v}') \geq 1$, since u and $\overline{v}'$ are non-zero, consequently

$$\operatorname{rank}(u\overline{v}') = 1$$