

The circle passes through the points (0,0) (0,5) (3,3) Find the equation of the circle.

The equation of the circle in general case is:

$$(x - a)^2 + (y - b)^2 = R^2$$

We have a system:

$$\begin{cases} a^2 + b^2 = R^2 \\ a^2 + (-5 - b)^2 = R^2 \\ (3 - a)^2 + (3 - b)^2 = R^2 \end{cases}$$

$$\begin{cases} a^2 + b^2 = a^2 + (5 + b)^2 \\ a^2 + (-5 - b)^2 = (3 - a)^2 + (3 - b)^2 \end{cases}$$

$$\begin{cases} b^2 - 25 - 10b - b^2 = 0 \\ a^2 + (5 + b)^2 = (3 - a)^2 + (3 - b)^2 \end{cases}$$

$$\begin{cases} b = -2.5 \\ a^2 + 6.25 = 9 - 6a + a^2 + 0.25 \end{cases}$$

$$\begin{cases} b = -2.5 \\ a = 0.5 \\ R^2 = 6.25 + 0.25 = 6.5 \end{cases}$$

$$R = \sqrt{6.25 + 0.25} = 2.55$$

$$(x - 0.5)^2 + (y + 2.5)^2 = 6.5$$