We prove that a is a unit of infinite order in ZG.

$\varphi : \mathrm{Z}G \to R$ defined by $\varphi(x) = \zeta$.. Note that $\varphi(a) = -2(1 + \zeta + \zeta + \zeta^3) - \zeta^3 - 3\zeta^2 - \zeta + 2 = -\zeta(3 + 5\zeta + 3\zeta^2)$. On the other hand, $(1 + \zeta)^4 = 1 + 4\zeta + 6\zeta^2 + 4\zeta^3 - (1 + \zeta + \zeta^2 + \zeta^3) = \zeta(3 + 5\zeta + 3\zeta^2)$. Therefore, $\varphi(-a(xu)^{-2}) = 1$. Since $-a(xu)^{-2}$ has clearly augmentation 1, our earlier argument implies that $-a(xu)^{-2} = 1$. It follows that $a = -(xu)^2$ is a unit of infinite order in ZG, with $a^{-1} = -(xu)^{-2} = -x^3v^2 = -x^3(1 - x - x^4)^2 = -x^3(1 + x^2 + x^8 - 2x - 2x^4 + 2x^5) = 2x^4 - 3x^3 + 2x^2 - x - 1$.