Let $u = 1 - x2 - x3$, $v = 1 - x - x4$, $R = Z[\zeta]$ where ζ is a primitive 5th root of unity. By Dirichlet's Unit Theorem, U(R) has rank 1. To shorten the proof, we shall assume the number-theoretic fact that $1 + \zeta$ is a fundamental unit in R. Consider the map $\varphi : ZG \to R$ defined by $\varphi(x) = \zeta$. Then $uv = 1$ (by direct calculation), and $\varphi(xu) = \zeta(1 - \zeta^2 - \zeta^3) = \zeta - \zeta^3 + (1 + \zeta + \zeta^2 + \zeta^3) = 1 + 2\zeta + \zeta^2 = (1+\zeta)^2$. Of course, $1 + \zeta$ has infinite order. Then u has infinite order.