Since $A^p = B^p = I$ and $AB = BA$, given two maps define a representation $D : G \to \mathrm{GL}_2(K)$. We shall prove a slightly stronger statement: For any extension field $L \supseteq K$, D cannot be equivalent over L to any representation of G over k. Indeed, assume otherwise. Then there exists $U = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{GL}_2(L)$ which conjugates both A and B into $GL_2(k)$. By explicit matrix multiplication,

$$U^{-1}BU = e^{-1}\begin{pmatrix} * & td^2 \\ -tc^2 & * \end{pmatrix}, U^{-1}AU = e^{-1}\begin{pmatrix} * & d^2 \\ -c^2 & * \end{pmatrix}$$

where $e = \det U = ad - bc \in L^*$. Therefore, td^2e^{-1}, d^2e^{-1}, tc^2e^{-1}, c^2e^{-1} all belong to k. Now c , d cannot both be zero, since $U \in \mathrm{GL}_2(L)$. If $c \neq 0$, we have $t = tc^2e^{-1}/c^2e^{-1} \in k$. If $d \neq 0$, we have similarly $t = td^2e^{-1}/d^2e^{-1} \in k$. This gives the desired contradiction.