Here χ_i are the characters of the simple left kG-modules, where k is a splitting field for G of characteristic not dividing $|G|$. The proof depends on the expressions for the central idempotents in kG giving the Wedderburn decomposition of kG into its simple components. These central idempotents are given by $e_i = |G|^{-1} n_i \sum_{g\in G} \chi_{\mathrm{i}}\left(g^{-1}\right) g$. Now use the equations $e_i e_j = \delta_{ij} e_i$, where the δ_{ij} 's are the Kronecker deltas.

The coefficient of h^{-1} on the RHS is $\delta_{ij}\ |G|^{-1} n_i \chi_i(h)$, and the coefficient of h^{-1} on the LHS is $n_i n_j |G|^{-2} \sum_{g\in G} \chi_{\mathrm{i}}\left(g^{-1}\right) \chi_{\mathrm{j}}\left(hg\right)$. Therefore, the desired formula follows.