Since kG is semisimple, $|G|$ is a unit in k. Therefore, for any given $H \subseteq G$, we can form the element $e = |H|^{-1} \sum_{h \in H} h \in kH \subseteq kG$.

We have $e^2 = |H|^{-2} \sum_{h \in H} h = e$, so e is an idempotent. Writing $kG = D_1 \times \cdots \times D_r$ where the D_i's are division rings, we see that any idempotent in kG is central. Therefore, for any $g \in G$, we have $geg^{-1} = e$. This implies that $gHg^{-1} = H$, so $H \triangleleft G$.