

Let G be the abelian group G/G' . Then kG is a commutative ring, so the natural ring homomorphism $\varphi_0 : R \rightarrow kG$ sends $ab - ba$ to zero, for all $a, b \in R$. This shows that $\varphi_0(I) = 0$, so φ_0 induces a k -algebra homomorphism $\varphi : R/I \rightarrow kG$. Next, we shall try to construct a k -algebra homomorphism $\psi : kG \rightarrow R/I$. Consider the group homomorphism $\psi_0 : G \rightarrow U(R/I)$ defined by $\psi_0(g) = g + I$. Since $\psi_0(gh) = gh + I = hg + I = \psi_0(hg)$ ($\forall g, h \in G$), ψ_0 induces a group homomorphism $G \rightarrow U(R/I)$, which in turn induces a k -algebra homomorphism $\psi : kG \rightarrow R/I$. It is easy to check that ψ and φ are mutually inverse maps, so we have $R/I \sim kG$.