

Next, let G_1 be the Klein 4-group $\langle a \rangle \times \langle b \rangle$. Here, $QG_1 \sim \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q}$, and the (central) idempotents giving this decomposition are determined again from the irreducible characters:

$$e_1 = (1 + a + b + ab)/4,$$

$$e_2 = (1 + a - b - ab)/4,$$

$$e_3 = (1 - a + b - ab)/4,$$

$$e_4 = (1 - a - b + ab)/4.$$

Finally, let G_2 be the quaternion group $\{1, i, j, k, \varepsilon, \varepsilon i, \varepsilon j, \varepsilon k\}$. Since there is an irreducible QG_2 -module given by the rational quaternions H_0 , and four 1-dimensional QG_2 -modules, we have $QG_2 \sim \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q} \times H_0$. For the corresponding central idempotents, we can first construct the ones associated with the linear characters:

$$e_1 = (1 + \varepsilon + i + \varepsilon i + j + \varepsilon j + k + \varepsilon k)/8,$$

$$e_2 = (1 + \varepsilon - i - \varepsilon i + j + \varepsilon j - k - \varepsilon k)/8,$$

$$e_3 = (1 + \varepsilon + i + \varepsilon i - j - \varepsilon j - k - \varepsilon k)/8,$$

$$e_4 = (1 + \varepsilon - i - \varepsilon i - j - \varepsilon j + k + \varepsilon k)/8.$$

The last central idempotent can then be obtained as:

$$e_5 = 1 - e_1 - e_2 - e_3 - e_4 = (1 - \varepsilon)/2.$$