Since the three irreducible complex representations of $G = S_3$ can be defined over Q, we know that Q is a splitting field for G. The dimension equation $|G| = \Sigma \chi_i(1)^2$ here is $6 = 1^2 + 1^2 + 2^2$, so we have $QG \sim Q \times Q \times M_2(Q)$.
The three central idempotents of QG giving such a decomposition can be computed from the character table of S_3.
They are:
$e1 = \{(1) + (123) + (132) + (12) + (23) + (13)\}/6$,
$e2 = \{(1) + (123) + (132) - (12) - (23) - (13)\}/6$,
$e3 = 1 - e1 - e2 = (1) * 2/3 - (123)/3 - (132)/3$.