

Since the three irreducible complex representations of $G = S_3$ can be defined over \mathbb{Q} , we know that \mathbb{Q} is a splitting field for G . The dimension equation $|G| = \sum \chi_i(1)^2$ here is $6 = 1^2 + 1^2 + 2^2$, so we have $\mathbb{Q}G \sim \mathbb{Q} \times \mathbb{Q} \times M_2(\mathbb{Q})$.