

if $2^x=3^y=12^z$ then $x+2y/xy = ?$ without using log

First:

$$\begin{aligned}2^x &= 3^y = 12^z \\3^y &= 12^z \\3^y &= 2^{2z} \cdot 3^z \\2^{2z} &= \frac{3^y}{3^z} \\2^{2z} &= 3^{y-z}\end{aligned}\tag{1}$$

Second:

$$\begin{aligned}2^x &= 3^y \quad | \text{ multiply by } 2z \\2^{2xz} &= 3^{2yz}\end{aligned}$$

Use equation (1):

$$\begin{aligned}3^{(y-z)x} &= 3^{2yz} \\(y-z)x &= 2yz \\x &= \frac{2yz}{y-z}\end{aligned}\tag{2}$$

Third:

$$\frac{x+2y}{xy} = \frac{1}{y} + \frac{2}{x}$$

Use equation (2):

$$\begin{aligned}\frac{x+2y}{xy} &= \frac{1}{y} + \frac{2}{\frac{2yz}{y-z}} \\ \frac{x+2y}{xy} &= \frac{1}{y} + \frac{y-z}{yz}\end{aligned}$$

Reduce right side to a common denominator:

$$\begin{aligned}\frac{x+2y}{xy} &= \frac{z}{yz} + \frac{y-z}{yz} \\ \frac{x+2y}{xy} &= \frac{z+y-z}{yz} \\ \frac{x+2y}{xy} &= \frac{y}{yz} \\ \frac{x+2y}{xy} &= \frac{1}{z}\end{aligned}$$