First consider the special case when $M \sim R^n$. In this case, we have the canonical isomorphisms
$(\mathrm{Hom}_R(M,N))^K \sim (\mathrm{Hom}_R(R^n,N))^K \sim (nN)^K \sim n \cdot N^K$,
$\mathrm{Hom}_{RK}(M^K,N^K) \sim \mathrm{Hom}^{RK}((R^K)^n,N^K) \sim n \cdot N^K$.
From this, we can safely conclude that θ is an isomorphism. (Some commutative diagrams must be checked, but it is mostly routine work.) Next we assume M is a finitely presented R-module, which means that there exists an exact sequence of R-modules $M1 \to M2 \to M \to 0$ where $M_1 = R^n$ and $M_2 = R^m$. Applying the left-exact Hom - functors into N and into N^K, we have the following commutative diagram:

$$\begin{array}{ccccccc} 0 & \to & \left(Hom_R\left(M,N\right)\right)^K & \to & \left(Hom_R\left(M_2,N\right)\right)^K & & \left(Hom_R\left(M_1,N\right)\right)^K \\ & & \downarrow\theta & & \downarrow\theta_2 & & \downarrow\theta_1 \\ 0 & \to & Hom_{R^K}\left(M^K,N^K\right) & \to & Hom_{R^K}\left(M_2^K,N^K\right) & \to & Hom_{R^K}\left(M_1^K,N^K\right) \end{array}$$

Since θ_1, θ_2 are both isomorphisms, an easy diagram chase shows that θ is also an isomorphism.