The “only if” part is clear. For the “if” part, assume every simple (left) R^L-module “comes from” a simple R^K-module. Let $M1, \dots, Mn$ be all the distinct simple R^L-modules, and let $M_i = U^L{}_i$, where $U_1, \dots, U_n$ are (necessarily distinct) simple R^K-modules. If there is another simple R^K-module V not isomorphic to any U_i, then, a composition factor of V^L would give a simple R^L-module not isomorphic to any M_i, a contradiction. Thus, $\{U_1, \dots, U_n\}$ give all the distinct simple R^K-modules. Each U_i remains irreducible over L and hence over the algebraic closure of L. Therefore, each U_i is absolutely irreducible, and we have proved that K is a splitting field for R.