For $a \in T(R)$, write $a = b + c$ where $b \in \operatorname{rad} R$ and $c \in [R,R]$. We have for every m: $b^{p^m} = (a-c)^{p^m} \equiv a^{p^m} - c^{p^m} \equiv a^{p^m}$ (mod $[R,R]$). Choosing m to be large enough, we have $b^{p^m} = 0$ (since rad R is nil). Therefore, the above congruence shows that $a^{p^m} \in [R,R]$. Now assume k is a splitting field for R, and let $a \in R$ be such that $a^{p^m} \in [R,R]$ for some m. Let $R' = R/\operatorname{rad} R \sim \prod_i A_i$ where $Ai = M_{ni}(k)$. Our job is to show that $a = a + \operatorname{rad} R$ belongs to $[R,R]$. Using the direct product decomposition above, we are reduced to showing that, for any i, the image $a'_i \in A_i$ of a' belongs to $[A_i,A_i]$ (given that $a_i'^{p^m} \in [A_i,A_i]$ for some m). Therefore, we may as well assume that $R = M_n(k)$. Here, let us compare $T'(R) := \{a \in R : a^{p^m} \in [R,R]$ for some $m \geq 1\}$ with $T(R) = [R,R]$. Then $T'(R)$ is a k-subspace of R containing $T(R)$. But, $T(R)$ has codimension 1 in R. Since $a = \operatorname{diag}(1, 0, . . . , 0) \notin T'(R)$, we must have $T(R) = T'(R)$.