Without any assumptions on k, the nilpotency of $(\text{rad } R)^K$ implies that $(\text{rad } R)^K \subseteq \text{rad}(R^K)$. Now assume R splits over k. The quotient ${}_R(R/\text{rad } R)$ is a semisimple R-module. Since every simple R-module remains a simple R^K-module upon scalar extension to K, $(R/\text{rad } R)^K$ remains a semisimple R^K-module. Therefore, $\text{rad}(R^K)$ annihilates $(R/\text{rad } R)^K = R^K/(\text{rad } R)^K$, which amounts precisely to $\text{rad }(R^K) \subseteq (\text{rad } R)^K$.