

Fix a composition series  $0 = M_0 \subset M_1 \subset \dots \subset M_n = M$  for  $M$ . Then  $0 = M^K_0 \subset M^K_1 \subset \dots \subset M^K_n = M^K$  can be refined into a composition series for  $M^K$ . Therefore, if  $\{V_i\}$  are the composition factors of  $M$ , then those of  $M^K$  are the composition factors of  $\{V^K_i\}$ . This gives immediately the “if” part, and the “only if” part is obvious.