For any endomorphism $f : k^n \to k^n$, im(f) is finitely generated. By the last exercise, im(f) is a direct summand of kn, and hence a projective k-module. This implies that ker(f) is finitely generated, and hence also a direct summand of k^n. Then it follows that $M_n(k) \sim End_k(k^n)$ is von Neumann regular. Conversely, suppose $Mn(k)$ is von Neumann regular for all $n \geq 1$. To solve the last exercise, we may again restrict ourselves to the case where M is a finitely generated (right) k-submodule of $P = k^n$. By enlarging n if necessary, we may assume that M can be generated by n elements. Thus M = im(f) for some $f \in End_k(k^n) \sim M_n(k)$. Since $M_n(k)$ is von Neumann regular, then M is a direct summand of k^n.