Say $M \subseteq P$ are right k-modules. Clearly, it suffices to handle the case where P is free with a *finite* basis $e_1, \ldots, e_n$. In this new situation, we carry out the proof by induction on n, the case $n = 1$ is obvious.

For $n \geq 2$, let $P_0 = e_1k \oplus \cdots \oplus e_{n-1}k$, and $M_0 = M \cap P_0$. By taking the projection $\pi : P \to e_nk$, we get a short exact sequence $0 \to M_0 \to M \to I \to 0$, where $I = \pi(M)$. By the beginning case of the induction (applied to the finitely generated submodule $I \subseteq e_nk$), we have $e_nk = I \oplus J$ for some k-submodule $J \subseteq e_nk$. Thus, I is projective. Hence short exact sequence splits, and M_0 is finitely generated. By the inductive hypothesis, $P_0 = M_0 \oplus N$ for some k-module N. Since $P_0 + M = P_0 + I$, we have now $P = P_0 \oplus I \oplus J = (P_0 + M) \oplus J = M \oplus (N \oplus J)$, as desired.

Since M is direct summand of a free module then it is projective.